

Efimov-like Resonances in Planar QED

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ABSTRACT: It is shown that planar topological effective gauge theory with dynamics, acquires corrections to angular momentum beyond the well-known topological photon spin, the latter arising from interactions with parity-breaking massive fermions. In the non-relativistic limit, a first quantized Schrödinger representation is possible where the topological and kinetic terms decouple, the latter contributing to angular momentum and compete with the centrifugal barrier. This results in shallow resonances of Efimov kind, which may be verified in planar physical systems.

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1 Introduction

Planar (2+1 dimensional) quantum electrodynamics (QED) displays enhanced renormalizability [1], tractable infrared behavior [2] and inherent topology [3–6]. The last property makes the theory relevant to phenomena as fractional Hall effect [7, 8], topological insulators [9, 10], anyon superconductivity [11, 12] and many more. The corresponding physics is effectively captured by the induced Chern-Simons (CS) term,

$$\mathcal{L}_{CS} := \frac{\mu}{2} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho,$$

with gauge field a_μ , imbuing the planar photon with spin $\mu/|\mu|$ [4, 6], though it does not effect dynamics of the gauge field [13]. Such an effective theory supports excitonic states [14] in presence of quantum fluctuations, which can survive the additional gauge dynamics [15, 16], even at finite temperature [16].

The fact that planar fermions can have non-trivial properties in presence of gauge interaction have been realized extensively [3–5, 14, 17], which can be expressed as an effective theory in the gauge sector [14, 16]. The latter approach is essentially a linear response treatment, characterized by the inherent topological aspects of the system, protected beyond first order quantum corrections by the celebrated Coleman-Hill theorem [18].

An effective gauge theory, capturing linear response of fermions, collectively represents the dynamics of the latter species. The CS term carries the signature of the parity-breaking fermion mass m , in addition to the usual transverse vacuum polarization contribution $\propto F_{\mu\nu} F^{\mu\nu}$ adding to the dynamics of the gauge field. A gauge theory, entirely defined by the CS term, was shown to satisfy the Schrödinger symmetry [14], and thereby is separable into spatial and temporal sectors independently [17]. When pure CS term is present at the *tree level*, the inherent topology imbibed into the fermion mass, together give rise to the parity anomaly [14], expressed as a correction to the fermion angular momentum [14, 17].

The effect on gauge angular momentum, which is gauge dependent [19], due to coupling with parity-breaking massive fermions in a plane, has not been evaluated yet. However, at the level of effective theories, which become important in case of linear response studies of properties like Hall effect [20], conductance [21] and spin control [15], such an analysis become important. Moreover, study of gravitational models in planar systems [22, 23], wherein gravity is equivalent to CS gauge fields [24], symmetries of the latter are of deep interest. More importantly, the inherent Lorentz symmetry of this system includes the CS contribution, emerging through interaction. The corresponding physical analogues, like graphene and topological insulators (TIs), are low-energy systems [21, 25], validating a linear response treatment.

In the following, the effect of quantum correction to the orbital angular momentum (OAM) of gauge field will be evaluated in Section 2. The corresponding topological influence will be marked out, the latter contributing to the gauge spin. In a non-relativistic approximation in suitable gauge in Section 3, will lead to a ‘Schrödinger equation of photon’, with *local* shift to the centrifugal potential, leading to shallow resonances with Efimov signature. This owes solely to the tree-level dynamics, and is devoid of topological effects or quantum fluctuations, complementing Coulomb interaction. Then we conclude in Section 4, suggesting possible physical realizations of such theories.

2 Corrections to angular momentum

Here we consider an effective gauge theory, with both dynamics and CS topology at the tree level, along with a vacuum polarization term representing the linear response of the interactions. The corresponding Lagrangian has the form,

$$\mathcal{L} \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mu}{2}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + \frac{1}{2}a_\mu\Pi^{\mu\nu}a_\nu, \quad (2.1)$$

where, the momentum space representation of the vacuum polarization tensor is [26],

$$\begin{aligned} \Pi^{\mu\nu}(q) &= \Pi_e(q)(q^\mu q^\nu - g^{\mu\nu}q^2) + \Pi_o(q)\epsilon^{\mu\nu\rho}q_\rho, \\ \Pi_e(q) &= \frac{e^2}{4\pi} \left[\frac{1}{|q|} \left(\frac{1}{4} + \frac{m^2}{q^2} \right) \log \left(\frac{2|m|+|q|}{2|m|-|q|} \right) - \frac{|m|}{q^2} \right], \\ \Pi_o(q) &= -i \frac{m}{4\pi} \frac{e^2}{|q|} \log \left(\frac{2|m|+|q|}{2|m|-|q|} \right). \end{aligned} \quad (2.2)$$

The corresponding energy-momentum tensor can be evaluated, generically, as the canonical conjugate to the metric tensor $g^{\mu\nu}$, from the following variation of the Lagrangian,

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} (\sqrt{-g}\mathcal{L}) \equiv 2 \frac{\delta\mathcal{L}}{\delta g_{\mu\nu}} + g^{\mu\nu}\mathcal{L}, \quad (2.3)$$

which is symmetric by construction [27]. For the present Lagrangian, the energy-momentum tensor has the form,

$$T^{\mu\nu} \equiv -F^{\mu\alpha}F^\nu{}_\alpha + \mu \left[\epsilon^{\mu\alpha\beta} a^\nu \partial_\alpha a_\beta + \epsilon^{\alpha\mu\beta} a_\alpha \partial^\nu a_\beta + \epsilon^{\alpha\beta\mu} a_\alpha \partial_\beta a^\nu + (\mu \leftrightarrow \nu) \right] + 2 \left[a^\mu \Pi^{\nu\alpha} a_\alpha + (\mu \leftrightarrow \nu) \right] + 2a_\alpha V^{\alpha\beta} \partial^\mu \frac{1}{\square} \partial^\nu a_\beta 2g^{\mu\nu} \mathcal{L} - 2\xi(\partial \cdot a) [\partial^\mu a^\nu + (\mu \leftrightarrow \nu)]; \quad (2.4)$$

with,

$$V^{\alpha\beta}(q) = P^{\alpha\beta} \left[\frac{|m|}{\pi(4m^2 - q^2)} \left(\frac{1}{4} + \frac{m^2}{q^2} \right) + i \frac{2m}{q^2} \Pi_o(q) - \Pi_e(q) \right] + \epsilon^{\alpha\rho\beta} q_\rho \left[i \frac{m|m|}{\pi(4m^2 - q^2)} - \Pi_o(q) \right]; \quad (2.5)$$

where,

$$P^{\mu\nu}(q) = q^\mu q^\nu - \eta^{\mu\nu} q^2,$$

in the momentum representation, in the covariant R_ξ gauge. At low energies, the form factors $\Pi_{e,o}$ can be considered as finite constants [2–6, 14].

The OAM tensor for any field is defined as [27],

$$\mathcal{M}^{\mu\nu\rho} := \int d^2x (T^{\mu\rho} x^\nu - T^{\mu\nu} x^\rho),$$

leading to the OAM pseudo-vector in planar systems as,

$$J = \int d^2x \mathbf{x} \times \mathbf{T}, \quad T^k = T^{0k}, \quad (2.6)$$

with spatial indexes expressed in Latin. From Eq. 2.4, ‘spatial’ components of the energy-momentum tensor can be written as,

$$T^{0k} \equiv -F^{0i}F^k{}_i + \mu \left[\epsilon^{0ij} a^k \partial_i a_j + \epsilon^{i0j} a_i \partial^k a_j + \epsilon^{ij0} a_i \partial_j a^k + \epsilon^{k\alpha\beta} a^0 \partial_\alpha a_\beta + \epsilon^{\alpha k\beta} a_\alpha \partial^0 a_\beta + \epsilon^{\alpha\beta k} a_\alpha \partial_\beta a^0 \right] + 2a^0 \Pi^{k\alpha} a_\alpha + 2a^k \Pi^{0\alpha} a_\alpha + 2a_\alpha V^{\alpha\beta} \partial^0 \frac{1}{\square} \partial^k a_\beta - 2\xi(\partial \cdot a) (\partial^0 a^k + \partial^k a^0). \quad (2.7)$$

which finally leads to the evaluation of OAM of spatial component of the gauge field through the evaluation of the commutator $[J, a^i(x)]$ [17], which can be resolved into commutators of gauge field and its derivatives. The inherent gauge redundancy of the system makes the fundamental commutators dependent on the choice of the gauge. For the future convenience, we choose the ‘physical’ Coulomb gauge $\nabla \cdot \mathbf{a} = 0$, implementing the *spatial* transverse nature of the gauge field, leading to [28],

$$[\dot{a}^i(x), a^j(y)]_{x_0=y_0} = i\delta_{TR}^{ij}(\mathbf{x} - \mathbf{y}); \quad \text{where,} \quad (2.8)$$

$$\delta_{TR}^{ij}(\mathbf{x} - \mathbf{y}) = \delta^{ij} \delta(\mathbf{x} - \mathbf{y}) + \partial^i \frac{1}{\nabla^2} \partial^j \delta(\mathbf{x} - \mathbf{y}),$$

with rest of the combinations vanishing. The choice of a *non-relativistic* gauge is justified even though we have been explicitly employing the covariant R_ξ gauge till now, as we are dealing with the OAM, which is spatial. Equivalently, the particular covariant gauge choice can be considered to have a spatial subsector of the desired form. Finally, the OAM of the spatial gauge field component turns out to be,

$$\begin{aligned} [J, a^k] \equiv & -L \left[\eta^{\alpha k} - i \frac{1}{q^2} V^{\alpha k} \right] a_\alpha - i \{1 - i\Pi_e\} \left[\epsilon^{kj} a^j - \partial^k \{\mathbf{x} \times \mathbf{a}\} \right] \\ & - i \{\mu - i\Pi_o\} x^k a_0 + i\Pi_e \epsilon^{kj} x^j \dot{a}_0 + (\text{Higher derivative terms}). \end{aligned} \quad (2.9)$$

There are a few points to be noted here. The naïve tree-level OAM $L := -i\mathbf{x} \times \nabla_x$ gets modified by a term proportional to $V^{\mu\nu}$, the latter incorporating quantum corrections through $\Pi_{e,o}$, along-with contribution from tree-level dynamics. Both form factors are singular at the two-fermion threshold $q^2 = 4m^2$, as signature of the planar exciton [14, 16]. The next two terms represent anomalous contribution to the planar OAM. The second is the planar analogue of photon spin in 3+1 dimensions [19], whereas the third represents the topological spin of planar gauge fields, with both containing respective quantum corrections. The latter also corresponds to the parity anomaly of the fermion sector [14]. Both these terms are necessary for complete Poincaré invariance of the theory. Rest of the terms contain further quantum corrections and higher derivatives, which are sub-dominant at the long wavelength limit. As $V^{\mu\nu}$ contain off-diagonal contributions, there will be contribution from temporal component a_0 to the angular momentum of the spatial ones. Thus the above contains contribution to OAM of planar photon from both tree level dynamics and vacuum fluctuations.

3 Schrödinger Equation of Photon

Modification to OAM of the planar gauge field becomes considerably important provided a first quantized formulation of the system is possible. For a Lorentz covariant vector field, such a reduction is not possible in general, owing to the instability of the vacuum (*i.e.*, presence of negative norm states). However, in case of a gauge field, choice of a non-relativistic gauge can reduce its symmetry to a Galilean one [27]. That is how Maxwell field can couple to a non-relativistic electron. In both Coulomb and axial gauge, a first quantized equation of motion was obtained for planar QED with only the CS term at the tree level [17]. In such an equation, modification to OAM, as in Eq. 2.9, alters the centrifugal potential, effecting the radial dynamics of the ‘photon’.

Provided suitable gauge-fixing is adopted, an N -particle state can be defined in the effective gauge sector, stable without introduction of anti-particles, as [17],

$$|N\rangle = \int d^2x_1 \dots d^2x_N \phi^\dagger(x_1) \dots \phi^\dagger(x_N) f(x_1, \dots, x_N) |0\rangle,$$

where $f(x_1, \dots, x_N)$ is the N -particle wave-function. Crucial to this is the existence of a unique fermion mass operator, defined as,

$$M = m \int \psi^\dagger(x) \psi(x) d^2x, \quad (3.1)$$

accommodating the mass degeneracy, arising through Bargmann's superselection rule [29] in a harmless manner. In effect, M serves as a central extension to the proposed Schrödinger algebra [14], satisfying,

$$[M, \psi(x)] = -m\psi(x) \quad \text{and} \quad [M, a_i(x)] = 0 = [M, a(x)]. \quad (3.2)$$

For eigenstates $|N\rangle$ being the eigenstate of the N -particle Hamiltonian, with eigenvalue (energy) E , the corresponding N -particle Schrödinger equation can be written as,

$$Ef(x_1, \dots, x_N) = -\frac{1}{2m} \sum_{i=1}^N \left[\nabla_i + ie^2 \nabla'_i \sum_{j \neq i} \mathcal{D}(x_{ij}) \right]^2 f(x_1, \dots, x_N); \quad (3.3)$$

where, $x_{ij} = x_i - x_j$,

in Coulomb gauge [17]. Here, ∇'_i acts on the function immediately next to it and repeated indexes do *not* mean summation. The propagator $\mathcal{D}(x_i - x_j) \equiv \mathcal{D}(\mathbf{x}_i - \mathbf{x}_j)$ correspond to spatial ($\mathbf{a}(x)$) or temporal ($a_0(x) := a(x)$) gauge field components, representing dynamics of a *single* particle. The effective Lagrangian in Eq. 2.1 now can be resolved in terms of these spatial and temporal components as,

$$\begin{aligned} \mathcal{L} \equiv & \frac{1}{2} \{1 + \Pi_e\} \left[a \nabla^2 a + \mathbf{a} \cdot \nabla^2 \mathbf{a} + 2\dot{a} \nabla \cdot \mathbf{a} + (\nabla \cdot \mathbf{a})^2 + \dot{\mathbf{a}} \cdot \dot{\mathbf{a}} \right] \\ & + \frac{i}{2} \{\Pi_o + i\mu\} [a(\nabla \times \mathbf{a}) + \mathbf{a} \times \nabla a + \mathbf{a} \times \dot{\mathbf{a}}]; \end{aligned} \quad (3.4)$$

where, $a^\mu = (a, a^k)$, $\dot{x} := \partial_t x$.

Application of the non-relativistic Euler-Lagrange equation,

$$\partial_t \frac{\delta \mathcal{L}}{\delta \dot{\phi}} + \partial_k \frac{\delta \mathcal{L}}{\delta \partial_k \phi} = \frac{\delta \mathcal{L}}{\delta \phi},$$

leads to the respective equations of motion for spatial and temporal components as,

$$2\ddot{a}^i - \nabla^2 a^i - \partial_i (\nabla \cdot \mathbf{a}) = 2\partial_i \dot{a} + i2 \left[\frac{\Pi_o + i\mu}{1 + \Pi_e} \right] \epsilon^{ij} \partial_j a \quad (3.5)$$

$$\text{and} \quad \nabla^2 a = 2\nabla \cdot \dot{\mathbf{a}} - i2 \left[\frac{\Pi_o + i\mu}{1 + \Pi_e} \right] \nabla \times \mathbf{a}. \quad (3.6)$$

The gauge components are interlinked, even in the Coulomb gauge, reflecting the underlying 'actual' Lorentz symmetry, owing to the Maxwell term in the original Lagrangian. This elucidates the fundamental difference of the present theory with one with pure CS gauge term [17]. The fact that the gauge sector is *not* merely topological, dynamics of the effective

theory carries the signature of the complete symmetry of the same, the space-time sectors become interdependent. The dynamics of respective components are defined by the equation with Laplacian of the same, thereby leading to the respective propagators. However, this identification excludes the effect of quantum corrections to the respective propagators, thereby excluding the same from the ‘photon Schrödinger equation’. This is expected from a non-relativistic, first-quantized approach, with energies far lower than those required for the excitation of virtual pairs.

More intriguingly, the complete topological sector gets decoupled from the dominant dynamics, including the tree-level CS contribution, tagged by the coefficient μ . Unlike the case in Ref. [17], the gauge sector had a tree-level dynamics that overcame the inherently topological CS part that incorporates interaction, as seen in the last terms of both the Eq.s 3.5 and 3.6. As we will see this distinction will prevail as the anomalous contribution to OAM of the ‘effective’ photon.

The temporal component a satisfies the Laplace’s equation, yielding the ‘static’ propagator $\propto \log |\mathbf{x}_i - \mathbf{x}_j|$ in plane. The presence of dynamics, however, makes the corresponding result for the spatial counterpart more involved. The propagator is time-dependent in general, with the momentum-space representation,

$$G^{ij}(\omega, \mathbf{k}) = \frac{1}{2\omega^2 - \mathbf{k}^2} \left[2\delta^{ij} + \frac{k^i k^j}{\omega^2 - \mathbf{k}^2} \right]. \quad (3.7)$$

From Eq. 3.3, as the time-independent Schrödinger equation is under consideration, the analysis is to be confined to the spatial sector with \mathbf{a} having a fixed energy ω , the latter being a free parameter. This is *not* the single-particle energy, and defines an N -particle state with definite energy, represented by spatial component of the gauge field. This effectively projects the system onto the position-space, leading to the propagator,

$$G^{ij}(\omega, \mathbf{x} - \mathbf{y}) := \int \frac{d^2 k}{(2\pi)^2} G^{ij}(\omega, \mathbf{k}) \exp \{-i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y})\}.$$

The angular integral identities,

$$\int_0^{2\pi} \cos \{a \cos(\theta)\} = 2\pi J_0(|a|) \quad \text{and} \quad \int_0^{2\pi} \sin \{a \cos(\theta)\} = 0,$$

with J_n being the n -th order Bessel function of the first kind, leads to the final expression of the position space propagator for \mathbf{a} as,

$$\begin{aligned} G^{ij}(\omega, \mathbf{r}) \equiv & i \left[\frac{1}{4\omega r} \left\{ \sqrt{2}J_1(\sqrt{2}\omega r) - J_1(\omega r) \right\} - \frac{1}{2}J_0(\sqrt{2}\omega r) \right] \delta^{ij} + \frac{1}{4r^2} \left[J_0(\sqrt{2}\omega r) - J_2(\sqrt{2}\omega r) \right. \\ & \left. - \frac{1}{2}J_0(\omega r) + \frac{1}{2}J_2(\omega r) \right] r^i r^j - \frac{1}{4\omega r^3} \left\{ \sqrt{2}J_1(\sqrt{2}\omega r) - J_1(\omega r) \right\} r^i r^j; \end{aligned} \quad (3.8)$$

where, $\mathbf{r} := \mathbf{x} - \mathbf{y}$, $r := |\mathbf{r}|$.

The above expression considerably simplifies to,

$$G^{ij}(\omega, \mathbf{r}) \equiv -\frac{i}{2} J_0(\sqrt{2}\omega r). \quad (3.9)$$

in the Coulomb (or radiation) gauge, effectively considering the long-range contribution, given the present low-energy treatment.

The effect of modified OAM can most simply be shown for the two-particle case, with the dynamics given by Eq. 3.3, which has been reduced to the following form:

$$Ef_n(\mathbf{x}_1, \mathbf{x}_2) = -\frac{1}{2m} \sum_{(i,k)=1}^2 \left[\partial_i + ie^2 \epsilon^{pl} \sum_{j \neq i}^2 \partial_i^l G^{nm}(\mathbf{x}_{ij}) \right] \times \left[\partial_k + ie^2 \epsilon^{pq} \sum_{r \neq k}^2 \partial_k^q G^{ms}(\mathbf{x}_{kr}) \right] f_s(\mathbf{x}_1, \mathbf{x}_2); \quad (3.10)$$

with, $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$,

where the repeated indexes l, m, n, p, q, s are summed. The notation ∂_j^i denotes differentiation with respect to the i -th component of the j -th coordinate, and repeated indexes means summation. Then, usual identities for Bessel functions,

$$\frac{d}{dx} [x^m J_m(x)] = x^m J_{m-1}(x), \quad J_{-n}(x) = (-1)^n J_n(x)$$

and $\frac{d}{dx} J_m x = \frac{1}{2} [J_{m-1}(x) - J_{m+1}(x)],$

finally leads to the two-particle Schrödinger equation,

$$Ef_n(\mathbf{x}_1, \mathbf{x}_2) \equiv \frac{1}{2m} \left[\mathbf{p}_1^2 + \mathbf{p}_2^2 + \omega^2 e^4 \frac{J_1^2(\sqrt{2}\omega r)}{2r^2} + \frac{\omega e^2}{\sqrt{2}} \left\{ (\mathbf{x}_1 - \mathbf{x}_2) \times (\mathbf{p}_1 - \mathbf{p}_2), \frac{J_1(\sqrt{2}\omega r)}{r^2} \right\} \right] f_n(\mathbf{x}_1, \mathbf{x}_2), \quad (3.11)$$

valid for the spatial gauge component. Here, the curly bracket represents anti-commutator, representing the symmetric quantum product of two operators.

An equivalent effective single-particle equation can be obtained by adopting the center-of-mass (CM) variables [17]:

$$\mathbf{p}_1 := \frac{1}{2} \mathbf{P} + \mathbf{p}, \quad \mathbf{p}_2 := \frac{1}{2} \mathbf{P} - \mathbf{p}; \quad \mathbf{R} := \frac{1}{2} (\mathbf{x}_1 + \mathbf{x}_2), \quad \mathbf{r} := \mathbf{x}_1 - \mathbf{x}_2,$$

yielding, $f_n(\mathbf{x}_1, \mathbf{x}_2) = f_n(\mathbf{R}, \mathbf{r}) \equiv \exp(i\mathbf{P} \cdot \mathbf{R}) \exp(iL\theta) h_n^L(r),$ (3.12)

leading to the radial Schrödinger equation for the present system,

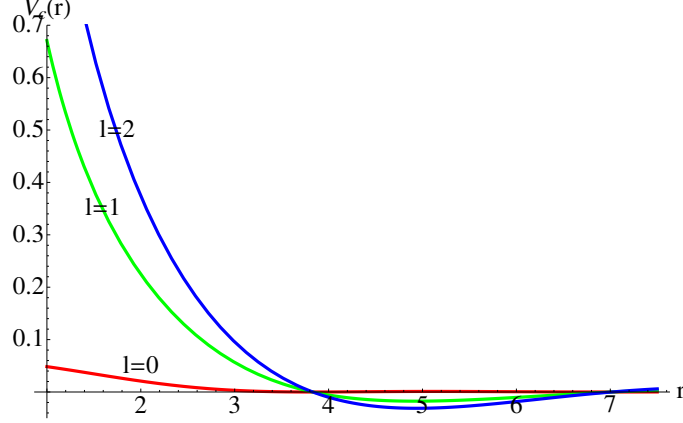


Figure 1. Plots of the correction to centrifugal potential $V_c(r)$ for different values of OAM: $L^2 = l(l+1)$, in the “Schrödinger equation” for photon. This shows possibility of a ‘centrifugal resonance’. Here, $\omega = 1/\sqrt{2}$ and $e = 1$. The presence of the centrifugal repulsion L^2/r^2 prevents the formation of a shallow bound-state.

$$2m\tilde{E}h_n^L(r) = \left[-\frac{1}{r} \frac{d}{dr} r \frac{d}{dr} + \frac{\left\{ L + \frac{\omega e^2}{\sqrt{2}} J_1(r) \right\}^2}{r^2} \right] h_n^L(r); \quad (3.13)$$

$$\tilde{E} := E - \frac{\mathbf{P}^2}{4m}, \quad J_1(r) := J_1(\sqrt{2}\omega r),$$

with \mathbf{P} being the CM momentum. The angular momentum is shifted by a local function, leading to a non-trivial centrifugal behavior, interpretable as effective mutual interaction between two *effective* spatial photons, mediated through interaction with low-energy fermions. The spatial photons are the effective description of fermions interacting via exchange of *actual* photons, which can very well be a fermion-anti-fermion bound state at sufficiently low energies [14, 16]. Therefore, this anomalous contribution to centrifugal interaction can very well be interpreted in terms of Coulomb interaction between charged fermions, mediated by photons. This modification to the centrifugal term is,

$$V_c(\omega r) := \omega^2 e^4 \frac{J_1^2(\sqrt{2}\omega r)}{2r^2} + \sqrt{2}L\omega e^2 \frac{J_1(\sqrt{2}\omega r)}{r^2}, \quad (3.14)$$

which is depicted in Fig. 1. This ‘shift’ display local minima for certain values of OAM, separated by shallow barriers. Such regions can capture a system for finite durations, thereby leading to possible resonances against the centrifugal repulsion L^2/r^2 , except for in the $l = 0$ channel (*s*-wave). As discussed before, this centrifugal extension is an effect of tree-level dynamics, devoid of topological contributions. However, this result is exclusive to 2+1 dimensions, as per eq. 3.9. Therefore, the dynamic and topological sectors separate out in the non-relativistic limit, the latter being known to yield constant shift to the OAM [17] in absence of tree-level dynamics, with a possible anyonic sector [30].

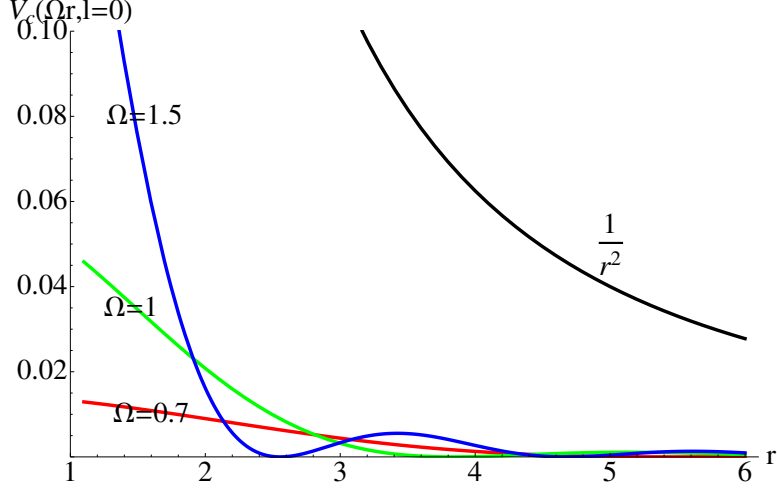


Figure 2. Plots of the s -wave centrifugal potential, for different values of $\Omega = \sqrt{2}\omega$, and the inverse-square potential, showing the prior to be of shorter range, required for the existence of Efimov-like states. Further, suitably high value of Ω is required to attain a local minimum, leading to resonance. Here $e = 1$.

Efimov connection: The local shift to the centrifugal term can be related with the Efimov physics [31], the latter being realized for three non-identical particles, with at least two of the three being almost bound. Then, two particle can experience shallow, near-resonance bound states with energies obeying constant scaling. The exact nature of the interaction is immaterial, as long as it is shorter in range than r^{-2} in radial coordinate r . In the present case, the s -wave centrifugal potential (Eq. 3.14) satisfies this condition, as shown in Fig. 2, for suitable values of gauge-field energy ω .

The total number of Efimov states up to energy $E \rightarrow 0$ can be expressed as the radial integral,

$$N_l(E) = \frac{m}{\pi} \int_{r_0}^a \sqrt{E - V_l(r)} dr, \quad \hbar = 1. \quad (3.15)$$

with r_0 being the range of interaction, l labeling the particular angular momentum channel and a being the scattering length at ($a \rightarrow \infty$). At very low energies, only the s -channel ($l = 0$) is of importance. The corresponding potential is,

$$V_l(r) = \frac{1}{2m} \frac{l(l+1) - s_0^2}{r^2}, \quad (3.16)$$

leading to $V_0(r) = -s_0^2/2mr^2$, under the WKB approximation. This finally yields the result $N_0 \cong (s_0/\pi) \log(a/r_0)$. Here, s_0 is a constant parameter to be tuned suitably. The full s -wave ($l = 0$) centrifugal potential, corresponding to low energies, becomes,

$$V_{l=0}(r) = -\frac{1}{2m} \frac{s_0^2}{r^2} + \omega^2 e^4 \frac{J_1^2(\sqrt{2}\omega r)}{2r^2}. \quad (3.17)$$

As we are interested in the long-range (asymptotic) behavior, the centrifugal potential are sub-dominant to the first term, as can be seen from Fig. 2. This leads to the expression for ground-state degeneracy as,

$$N_{l=0}(E=0) = \frac{1}{2\pi} \sqrt{\frac{m}{2}} \int_{r_0}^a \frac{dr}{r}, \quad (3.18)$$

leading to the exact result for the Efimov case. Therefore, the non-relativistic, first-quantized, effective treatment of the planar gauge field, interacting with fermions, leads to Efimov-like *resonances*. However, having this interaction as an inherent three-body effect is not that straight-forward, though an effective theory may be constructed.

4 Discussion and Conclusion

The quantum corrections to OAM, both due to tree-level dynamics of the gauge field and vacuum fluctuations, carry the relativistic signature, reflected by the interdependence of spatial and temporal components. This re-affirms the gauge-dependence of spin part of the total angular momentum of the gauge field [19], requiring suitable gauge fixing. More importantly, quantum fluctuations contribute in a singular way near the two-fermion threshold, physically representing the process of two fermions coming close together and thereby, contributing divergently to the angular momentum of the effective degree of freedom. This is in conformity with the excitonic states in planar QED [14, 16].

The modification to the angular momentum in the effective gauge channel due to tree-level dynamics is reflected in the first-quantized approximation, though the same is not true for the contribution from quantum corrections, as expected. This non-relativistic approach is justified by the adopted Coulomb gauge [17], much like the interaction of electromagnetic field to non-relativistic charged particles. Interestingly, the tree-level topological (CS) part separates out and does not contribute to this modified OAM, unlike the case in Ref. [17]. The resultant Schrödinger equation of the photon displays shift in the OAM by a local function, thereby considerably modifying the centrifugal potential at low energies (s-wave), leading to shallow resonances of Efimov type. The fact that tree-level gauge dynamics alone can effect low-energy resonances may have deeper explanations [32].

Low-energy, effective planar QED is realizable in condensed matter systems like graphene [33] and topological insulators [21], with emergent gauge interactions [34]. The emergent Dirac fermions in these systems can allow for direct measurement of the modification to OAM, from Eq. 2.9, following the fact that at low energies ($q \rightarrow 0$), $\Pi_e \rightarrow 0$, and only the topological form factor Π_0 survives as a constant, representing the Hall conductivity [25]. However, the physical realization of the photon Schrödinger equation demands non-relativistic effective dynamics, and thereby requires different physical systems.

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